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THIRTY YEARS OF EXPERIENCE WITH THE WAVE EQUATION SOLUTION BASED ON THE METHOD OF CHARACTERISTICS.

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ABSTRACT.

The method of characteristics, as an accurate solution of the wave equation, has a long history of application. This paper will describe a brief history of the method and the author's experiences with the method of characteristics over a period of 30 years. Developments in the Netherlands, especially around the 1970's are described.

In the 1970's Heerema started to research the dynamic behaviour of soil during driving. HBG (Hollandsche Beton Groep) started the development of the Hydroblok impact hammer. The Hydroblok was a high tech hammer based on nitrogen cushioning. HBG extended the theoretical solution of the wave equation (method of characteristics) with a straightforward theoretical solution for the shaft friction along the pile.

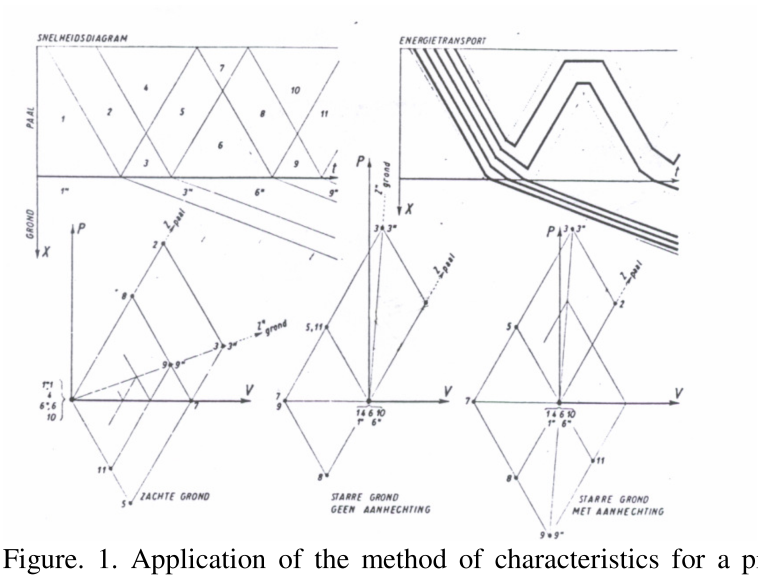


Figure. 1. Application of the method of characteristics for a pile with toe on sand (De Josseling de Jong ,1956)

The research institute TNO started the development of the wave equation program TNOWAVE in the late 1970's based on HBG's friction model extension for the method of characteristics. Today the program has a worldwide application in the field of pile testing. This paper will describe the method, its development and application over time and its relation to today's applications for Pile Driving Prediction (PDPWAVE) for impact hammers and vibratory hammers (VDPWAVE), signal matching for Pile Driving Analysis (DLTWAVE) or Dynamic Load Testing (DLT), Pile Integrity Testing Signal (SITWAVE) signal matching and Statnamic (STNWAVE) simulation

INTRODUCTION.

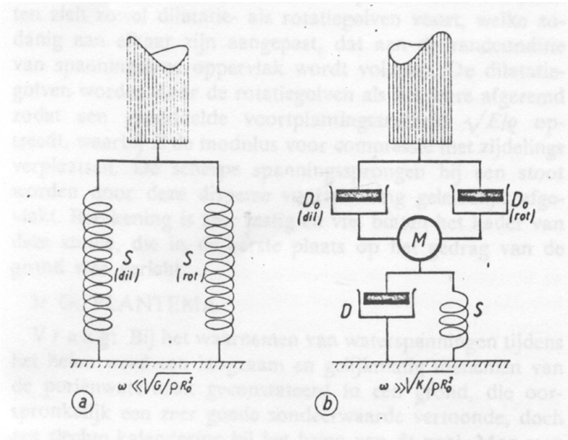


Figure 2. Toe resistance model for method of characteristics for low and high frequencies proposed by De Josseling de Jong (1956)

Although the stress wave theory and stress wave measurements were already applied in the Netherlands in the 1950's (Fig 1 and 2) (De Josselin De Jong, 1956) (Verduin, 1956), the field of stress wave applications got a real boost in the Netherlands when the Dutch companies Heerema, HBG and TNO intensified their research on this subject because of offshore pile driving activities on the North Sea for the oil industry.

Extending the work of Coyle and Gibson (1970) Heerema (1979) performed research at the TNO laboratories in 1975 to investigate the soil behaviour during driving to develop models for pile drive-ability analysis.

The company HBG (Hollandsche Beton Groep part of the HBM holding) started the development of the Hydroblok impact hammer in the Netherlands in the 1970's . The Hydroblok was a high tech hammer based on nitrogen cushioning to improve efficiency. The design of the hydroblok hammer was based on the theoretical solution of the wave equation (St. Venant , 1867), the so called method of characteristics. The method of characteristics was already used in the Netherlands for the prediction of the propagation of tidal waves, based on the work of Massau (1914) and Schonfeld (1951). De Josseling de Jong (1956) first applied the method of characteristics to pile driving and proposed a model for the toe resistance including porewater pressure phenomena.

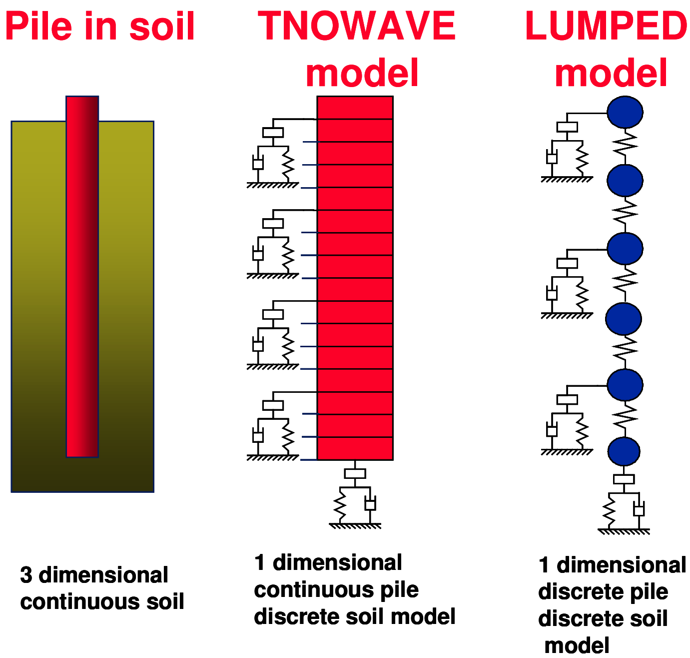


Figure 3 . Wave equation models

The original method of characteristics was valid for a free rod, not interacting with shaft friction or toe resistance. The method of characteristics is based on the phenomenon that stress waves in a frictionless pile propagate unaltered with a characteristic stress wave velocity. HBG extended the method to piles with a theoretical solution for the shaft friction (HBM method). Voitus van Hammer et all. published a paper (Voitus van Hammer et all, 1974) with the title “Hydroblok and Improved Pile Driving Analysis” on 2 May 1974 describing the above extended method of characteristics .

To the authors knowledge at that time closed theoretical solutions for the wave equation were only known for piles without shaft friction. When friction was introduced in the partial differential equation, this equation could only be solved analytically if the friction was present as an analytical function. The solution could only be found by integral transforms (such as the Laplace transform) and was obtained in the form of Fourier series (Van Koten, et al. 1980). It was practically impossible to find a solution, if for instance the shaft friction was assumed to depend on the velocity or displacement.

In those cases numerical integration of the differential equation could be done to provide the solution, as was already done at that time with Smith's wave equation program (Smith 1960). For the numerical integration method the mathematical model of the pile consists of a number of point masses (lumped model Fig. 3). The shaft friction and toe resistance are accounted for by a series of springs with dash-pots connected to the point masses.

Because of the disadvantages of the lumped mass approach, (see next sections), the HBG group has chosen a different approach. They started with the analytical solution valid for the frictionless case and applied the consideration that the continuous shaft friction could be replaced by a great number of concentrated frictional forces.

As stated by Voitus van Hamme: *When the friction is concentrated at a number of points the parts of the pile between these points are not subject to friction and the simple stress wave theory is valid for them.*

The discontinuities which occur at the point of action can be dealt with in a simple manner. For practical purposes the calculations had to be made by a computer and HBG developed their in house program PILEWAVE.

DEVELOPMENTS

At the end of the 1970's TNO wanted to develop its own wave equation program. There were three methods available for the algorithms:

- The finite difference method
- The finite element method
- The method of characteristics

It was known that the Smith algorithm gave approximate solutions and some major problems were known (van Weele, 1984):

- A falling ram with the same impedance as the pile did not result in a block shape wave in the pile as it should according to the theory

- A wave in a frictionless pile decreased in amplitude with depth because of numerical damping, while it should not. Furthermore the elements were not in rest after the wave front has passed, but still contained some energy, as they were vibrating.

- There was a limitation in the representation of higher frequencies. The higher the frequency, the larger the inaccuracies in the propagation of the wave.

- In the case of large end resistance the solution could become unstable.

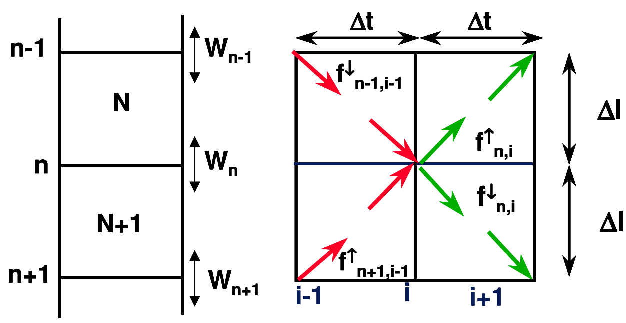
The algorithm proposed by HBG did not suffer from all these problems because it was based on the theoretical solution and not on an integration procedure. So TNO to adopt the method of characteristics because of :

- The elegance of the method
- The insight it gives in wave mechanics
- The lack of numerical stability problems

Other late starters also adopted the theoretical solution of the method of characteristics because of its accuracy. (Meunier, 1984.)

It should be mentioned that van Weele (van Weele and Kay, 1984) later proved that several of the problems of the Smith's algorithm could be overcome by modifications to the spatial and temporal discretisation of the program, by introducing the critical time-step.

Based on the HBG paper the development of the program TNOWAVE was started. The designed algorithm and applied soil models are presented in Figure 4.



$$f^{\downarrow}_{n,i} = \left(\frac{Z_N - Z_{N+1}}{Z_N + Z_{N+1}}\right) f^{\uparrow} + \left(\frac{Z_{N+1}}{Z_N + Z_{N+1}}\right)\left(2f^{\downarrow} - W_{n,i}\right) \quad (1)$$

$$f^{\uparrow}_{n,i} = \left(-\frac{Z_N - Z_{N+1}}{Z_N + Z_{N+1}}\right) f^{\downarrow} + \left(\frac{Z_N}{Z_N + Z_{N+1}}\right)\left(2f^{\uparrow} + W_{n,i}\right) \quad (2)$$

Figure 4 .TNOWAVE algorithm

$f^{\downarrow}$ = incident downward travelling wave at *n-1* and *i-1*,

$f^{\uparrow}$ = incident upward travelling wave at *n-1* and *i-1*,

$f^{\downarrow}_{n,i}$ = transmitted downward travelling force wave,

$f^{\uparrow}_{n,i}$ = transmitted upward travelling force wave,

n = discrete point or node number,

i = time step number,

Z_N = impedance of pile element N,

Z_{N+1} = impedance of pile element N+1,

N = pile element number.

W = soil interaction

$$W = W_u + W_v + W_a \quad (3)$$

Force: $f(n,i) = f^{\downarrow}_{n-1,i-1} + f^{\uparrow}_{n+1,i-1}$ (4)

Velocity : $v(n,i) = \frac{f^{\downarrow}_{n-1,i-1}}{Z_N} - \frac{f^{\uparrow}_{n+1,i-1}}{Z_{N+1}}$ (5)

Acceleration: $a(n,i) = (v(n,i) - v(n,i-1))/\Delta t$ (6)

Displacement: $u(n,i) = \sum v(n,i).\Delta t$ (7)

Power : $P(n,i) = f(n,i).v(n,i)$ (8)

Energy: $E = \sum P(n,i)$ (9)

The pile is divided into equally spaced intersections. The spaces between the intersections are often called elements, however they are not elements in the sense of finite element methods. The set of intersections common to each pair of spaces is to be considered as a co-ordinate system.

The waves arriving at the intersections are determined from the waves calculated at the former time-step. Between the inter-sections the pile is frictionless and so the propagation of a wave will be undisturbed. Arriving at the intersection a part of the wave will be transmitted and another part reflected.

The magnitude of transmitted and reflected waves depends on the pile properties and the shaft friction. The equations for transmitted and reflected waves are derived from the fulfilment of equilibrium and continuity conditions at the intersections and are calculated in a straightforward manner (Fig 5).

Force , velocity, displacement and accelerations can de calculated from the waves and allow the calculation of energy and friction.

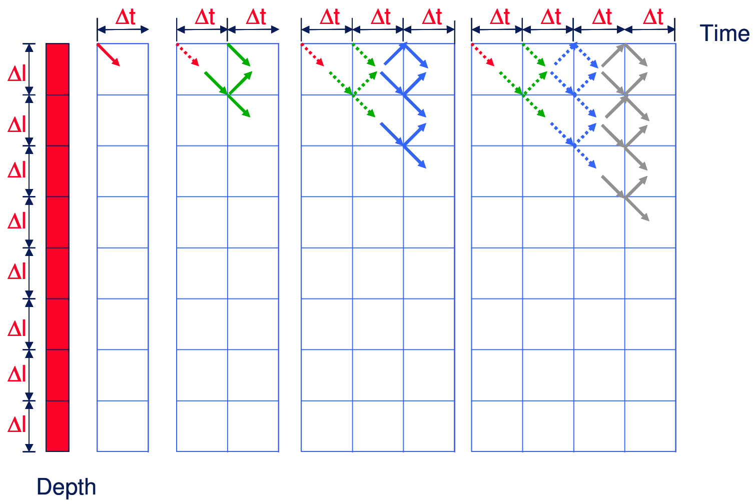


Figure 5. Successive propagation of waves in time/intersection co-ordinate system (Δl = distance between intersections, Δt = time step)

In the beginning the program was heavily tested to known and new theoretical solutions (van Koten, et al., 1980). Starting conditions like the velocity of the impacting ram (υ_{start}) are introduced by defining initial waves at the start of the calculation process.

$$v^{\downarrow}_{n,i=0} + v^{\uparrow}_{n,i=0} = v_{start} \qquad (10)$$

Additional external forces (F_{ext}) are introduced by defining force waves to take into account pre-stressing of the piles, additional pulling or compression forces during driving, or gravity. With Δm the mass of a pile part between two intersections and g gravity.

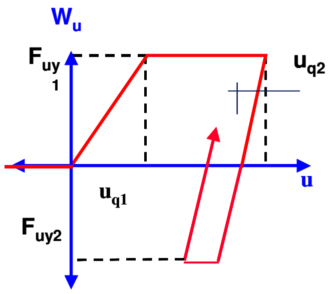
$$f^{\downarrow}_{n,i} + f^{\uparrow}_{n,i} = F_{ext} \qquad (11) \qquad\qquad f^{\downarrow}_{n,i} + f^{\uparrow}_{n,i} = \Delta m.g \qquad (12)$$

Parts representing the ram, anvil, striker plates, helmet, hammer-caps, inserts, and followers are separated by intersections that only transfer compression forces (no tension levels or splices, see also Fig.10). Hammer cushions and pile cushions are modelled as discrete springs or continuous parts. Diesel hammer combustion forces and Statnamic combustion forces are acting as external forces on the pile and ram or reaction mass. Vibratory hammer forces are also introduced as an external force on the pile top. With signal matching a measured force signal is introduced on the pile top as an external force.

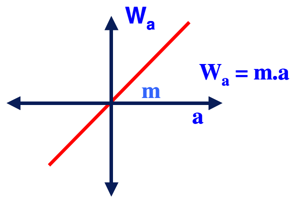
There are several possibilities to model the soil resistance but the most frequently applied are presented in Fig. 6. In principle three different types of soil resistance models are available: displacement dependent resistance W_u (springs), velocity dependent resistance W_v (dash-pots), and acceleration dependent resistance F_a (masses). The springs and dash-pots can be defined as linear and non linear.

$$W_u(n,i) = K_n.u(n,i) \qquad (13) \qquad\qquad W_v(n,i) = C_n.v(n,i) \qquad (14)$$

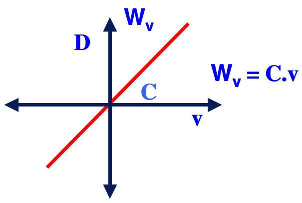
The acceleration-dependent force W_a, is treated differently by simply adding to the mass of the pile. The added mass interaction thus has an influence on the propagation velocity, as expected.



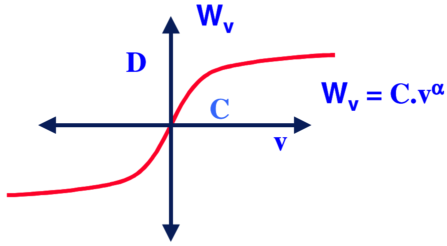
Displacement dependent soil model



Acceleration dependent soil model



Velocity dependent soil model



Velocity dependent soil model

Figure 6. Frequently applied soil models

For static analysis (load displacement diagrams) only the stiffness of the pile elements and soil springs are taken into account.

The damping factor C for the TNO Model is the damping factor as used in dynamics for dash-pots (Ns/m)

The total friction W_w acting at an intersection is

$$W_w = W_u + W_v + W_a \quad (15)$$

Other soil model parameters as provided by Smith, Rausche, Heerema and Randolph can be easily incorporated. For example if we look to damping factors.

For the damper resistance we can write

$$W_v = C.v \quad (16)$$

For the total friction

$$W_w = W_u + C.v + W_a \quad (17)$$

With Smith damping Js:

$$W_t = W_u\,(1 + J_s.v) = W_u + W_u.J_s.v = W_u + W_v \quad (18)$$

$$W_v = F_u.J_s.v \quad (19)$$

$$C = F_u.J_s \quad (20)$$

For the total friction

$$W_w = W_u + W_u.J_s.v + W_a \quad (21$$

With Case damping Jc:

$$W_v = J_c\,Z\,v \quad (22)$$

$$C = J_c\,Z \quad (23)$$

For the total friction

$$W_w = W_u + C.v + W_a \quad (24)$$

With Coyle and Gibson (Heerema)

$$W_t = W_u\,(1 + J_s.v^{\alpha}) = W_u + W_u.J_s.v^{\alpha} \quad (25)$$

$$W_v = F_u.J_s.v^{\alpha} \quad (26)$$

For the total friction

$$W_w = W_u + W_u.J_s.v^{\alpha} + W_a \quad (27)$$

In principal there is no difference between the damping factors and each can be expressed as a function of the other.

In the next stage of development the program was used for research into dynamic soil model parameters (Middendorp & van Brederode, 1984).In the early 1980's the TNOWAVE program was frequently used for offshore PDA applications to determine pile capacities and signal matching techniques similar to CAPWAP™ (Goble & Rausche, 1980) were developed.

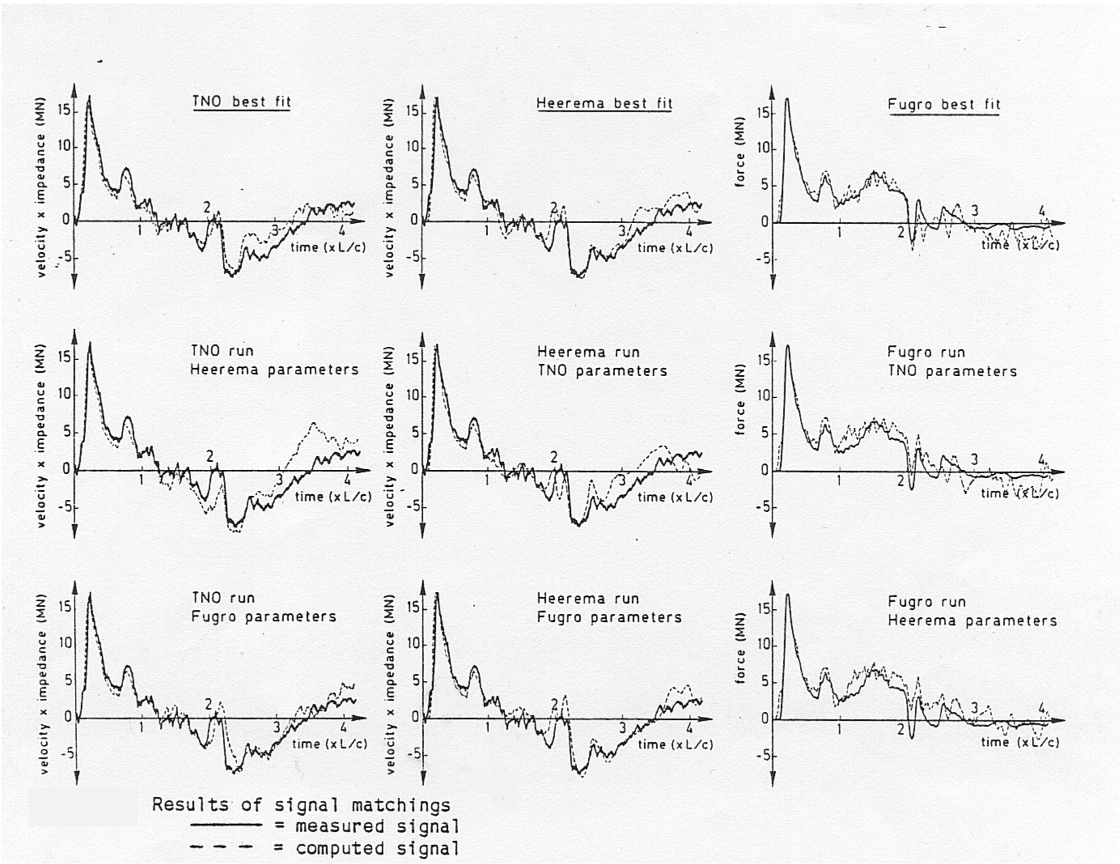


Figure 7. 1985 Comparison signal matching results, TNOWAVE (TNO), DYNPAC (Heerema) and CAPWAP (Fugro)

After gaining some experience the limitations of the signal matching techniques became apparent and were published (Middendorp & van Zandwijk, 1985). Signal matching results from CAPWAP, DYNPAC and TNOWAVE were compared. It became clear that the signal matching programs produce similar results when using the same input data. However it was also proven that the signal matching techniques did not yield unique solutions.

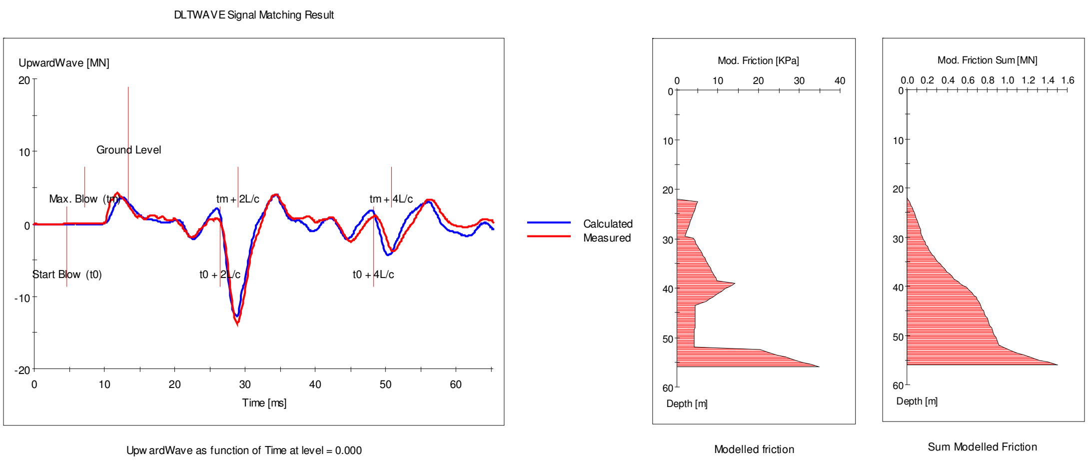


Figure 8. DLTWAVE signal matching results

The signal matching technique was also extended to pile integrity testing (Middendorp & Reiding, 1988). The technique was checked for piles with known discontinuities (Starke & Janes, 1988).

In the same period TNOWAVE was extended to vibratory pile driving prediction. At that time the theories to predict the performance of a vibro-hammer were base on a single lumped mass model for the pile. This schematisation is valid for relative short piles but not for long piles like offshore piles. A long pile is not moving as a single lumped mass, and stress wave phenomena have to be taken into account. With the method of characteristics applied to vibratory driving it could be proven to the offshore industry that long offshore piles could be installed with vibro-hammers (Jonker & Middendorp, 1988).

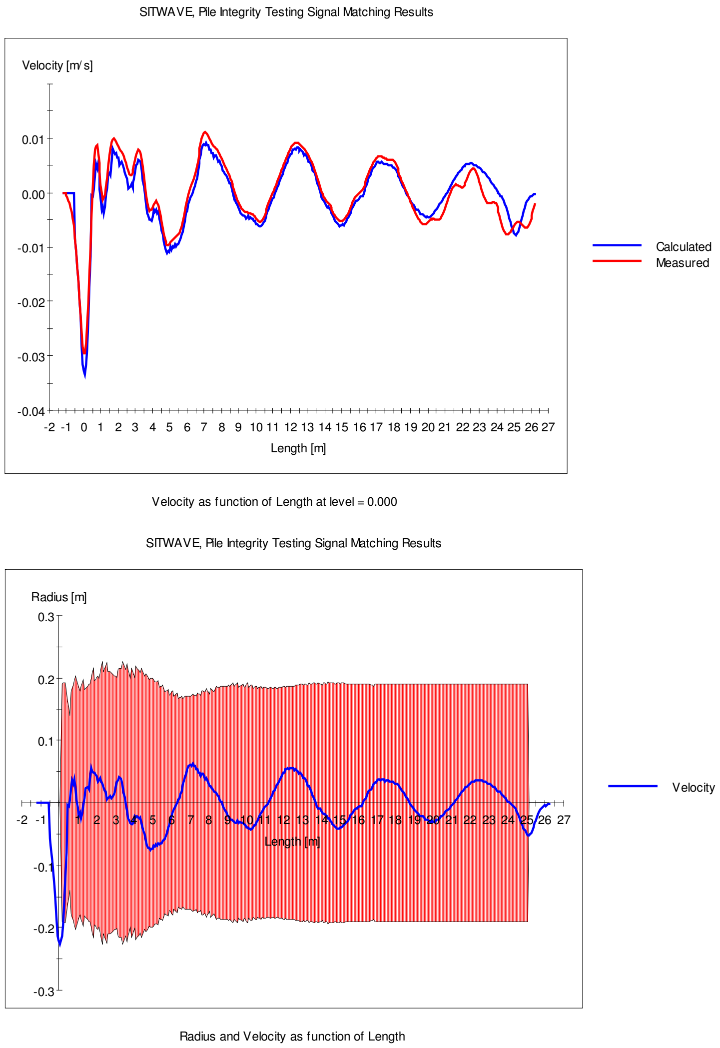


Figure 9. Pile Integrity Testing Signal Matching results

Although it is a small country the Netherlands has a huge piling market because of the soft soil conditions. Per year between 700 000 and one million pre-cast piles are driven and a similar amount of cast in situ piles are installed. To be competative in the market cost should be optimised. The Dutch pre-cast piling industry wanted to know the minimum amount of steel required for driving a pile without loss of quality and reliability. The main function of the reinforcement during driving is to prevent or to reduce cracking. To determine an optimum reinforcement ratio, the development of the steel stress at the cracks and the crack behaviour had to be known. A crack numerical model was developed and implemented into TNOWAVE (Bielefeld et al., 1988). The model contained the following options:

- Simultaneous wave propagation in the concrete and the reinforcement
- Bond forces between concrete and reinforcement
- Pre-stressing
- Cracking models
- Multiple cracking
- Opening and closing of cracks

From the late 1980's the author got strongly involved in the development of Statnamic (Bermingham & Janes, 1989). A method of analysis was developed based on the assumption that the for the relative long duration of the Statnamic load the pile could be modelled as a lumped mass and stress wave phenomena did not have to be taken into account. To check the validity of this statement the TNOWAVE program was extended to include Statnamic loading (Middendorp &, Bielefeld, 1995).

Because manual signal matching for PDA was time consuming, “Automatic” signal matching was developed (Courage & van Foeken, 1996) based on the Kalman filtering method. Although the method is called “Automatic” engineering judgement is still required to generate proper start values for the matching process and for the verification of the final results.

In 1999 TNOWAVE was converted to Windows and fully re-designed. An extensive hammer library was developed together with graphical modules for the input of soil investigation data, hammer energy levels and the drawing of pile shapes.

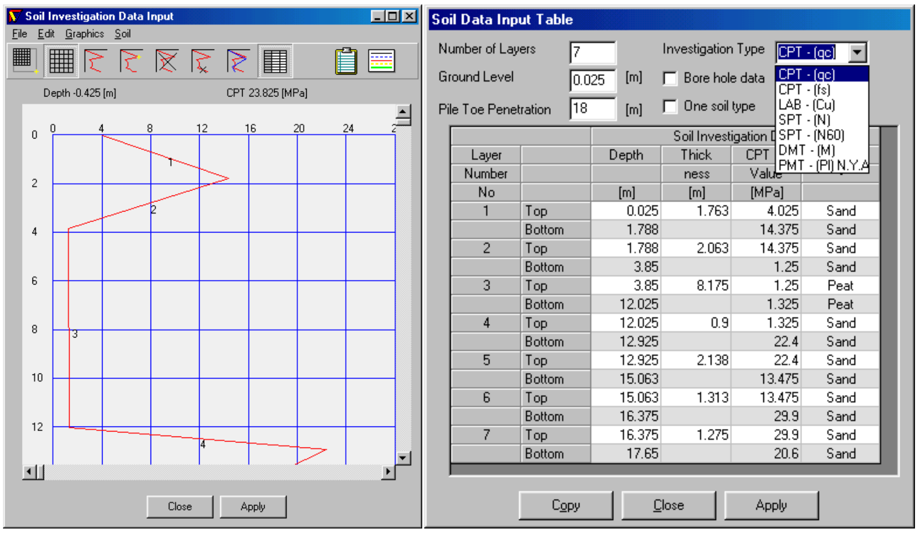


Figure 10. Soil investigation input module

The soils investigation module deals with many types of soils investigations like CPT, SPT, DMT and PMT or bore hole results. When soils investigation data is available on paper it is scanned into a digital file and the data can be digitised on the computer screen. In the Netherlands the soils investigation data like CPT results are available in a digital standard format GEF (Geotechnical Exchange Format) (CUR Report 1999). The CPT cone resistance and friction are directly transferred by reading the GEF file and the module automatically determines the soil types. In the next step the soils investigation data is converted automatically to the fundamental static and dynamic soil parameters as discussed above. Experienced users still have direct access to the fundamental static and dynamic soil parameters.

TNO further developed TNOWAVE applications for hammer and pile cushion optimisation (Jonker, van Foeken, 2000) and bottom drive casing in close co-operation with IHC.

RECENT DEVELOPMENTS

At present the TNOWAVE program is comprised of 5 modules. PDPWAVE for impact hammer pile driving prediction, VDPWAVE for vibratory driving prediction, STNWAVE for Statnamic simulation, SITWAVE for pile integrity signal matching and DLTWAVE for PDA and DLT signal matching.

A development initiated by hammer manufacturers is the distribution of simplified, limited, but user-friendly versions of PDPWAVE with libraries for their specific hammers and related components. Libraries for sheet piles and H-piles have been added. The limited versions present impact diagrams, blow count graphs and bearing graphs. At the moment versions have been developed for IHC from the Netherlands (IHCWAVE) and APE from the USA (APEWAVE). A version for the company Berminghammer from Canada (BERMWAVE) is under development.

The limited and simplified versions give a first indication about the drive-ability of a hammer-pile-soil combination.

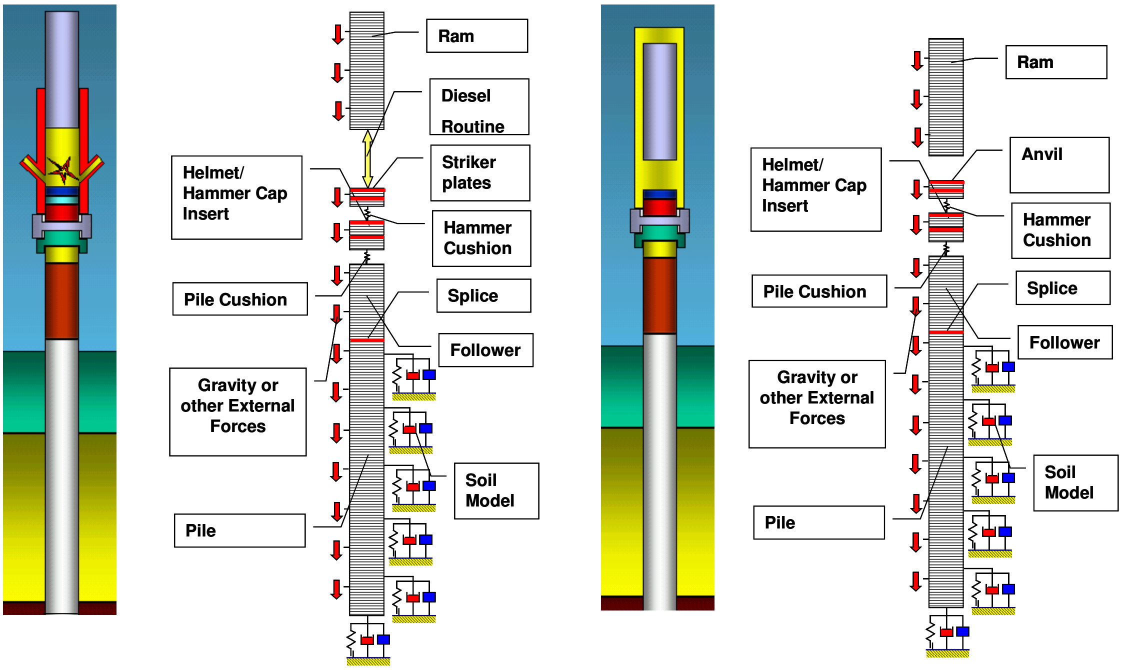


Figure 11. PDPWAVE models for for Diesel hammer and Hydraulic hammers

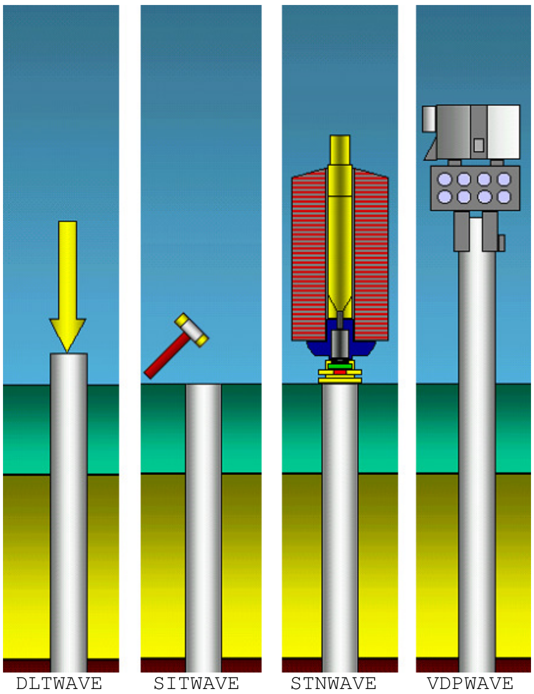


Figure 12. Additional components

The hammer manufacturer's intention is to distribute copies to their (potential) clients. For professional drive-ability studies, professional packages still have to be applied and verification by PDA measurements is required.

Since 1999 the pile testing and research activities of TNO have been privatised and transferred to the company Profound.

CONCLUSIONS

The application of the method of characteristics is based on a long history and is applied today as standard practice for a wide range of stress wave applications all over the world. It has evolved from a graphical research tool into a powerful, stable and reliable method for drive-ability studies, hammer design, signal matching techniques, and research and education.

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